Available online at https://www.rsu.ac.th/rjas

Rangsit Journal of Arts and Sciences, July-December 2015 RJAS Vol. 5 No. 2, pp. 187-197
Copyright © 2011, Rangsit University
ISSN2229-063X (Print)/ISSN2392-554X (Online)
DOI : 10.14456/rjas.2015.17

Synchronization of chaotic systems based on an interconnection and damping assignment-passivity-based control

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Submitted 5 February 2015; accepted in final form 20 April 2015
Available online 26 December 2015

Abstract

This paper presents the synchronizations of chaotic nonlinear systems. With the help of a passivity-based control design [interconnection and damping assignment passivity-based control (IDA-PBC)] method, a nonlinear control strategy is proposed to achieve the chaotic synchronization. In particular, there are two chaotic systems of interest, Genesio system and Chua's circuit system, which are employed as two examples for illustration. The simulations indicate the effectiveness and feasibility of the proposed method to synchronize the chaotic systems of interest. In addition, the performances of the proposed control scheme are evaluated and compared with an existing nonlinear control, in particular, backstepping controller.

Keywords: *chaotic synchronization, nonlinear control, interconnection and damping assignment passivity-based control, Genesio system, Chua's circuit system*.

บทคัดย่อ

บทความนี้นำเสนอการประสานกันของระบบไม่เป็นเชิงเส้นที่ไม่เป็นระเบียบ จากการใช้การออกแบบการควบคุมที่อาศัยการไม่มีปฏิกริยา (การควบคุมที่ไม่มีปฏิกริยาที่อาศัยการเชื่อมระหว่างกันและการกำหนดการหน่วง) เราจะได้ตัวควบคุมที่สามารถบรรลุถึงการประสานกันของระบบที่ไม่เป็นระเบียบ ระบบของ Genesio และระบบวงจรของ Chua เป็น 2 ตัวอย่างของระบบที่ไม่เป็นระเบียบที่ถูกนำมาใช้งานเพื่อแสดงถึงการประยุกต์ของวิธี IDA-PBC ที่นำเสนอ ผลการจำลองระบบชี้ให้เห็นถึงประสิทธิภาพและความเป็นไปได้ของวิธีการที่นำเสนอเพื่อให้เกิดการประสานกันของระบบไม่เป็นระเบียบข้างต้น นอกจากนี้สมรรถนะของวิธีการควบคุมที่นำเสนอจะถูกประเมินค่าและถูกเปรียบเทียบกับการควบคุมแบบก้าวถอยหลัง

คำสำคัญ: *การประสานที่ไม่เป็นระเบียบ, การควบคุมไม่เป็นเชิงเส้น, การควบคุมที่ไม่มีปฏิกริยาที่อาศัยการเชื่อมระหว่างกันและการกำหนดการหน่วง, ระบบของ Genesio, ระบบวงจรของ Chua*

1. Introduction

After Pecora and Carroll (Pecora & Carroll, 1990) introduced a method for chaotic synchronization, there have recently been considerable interests in investigating the synchronization of a variety of chaotic systems. Chaotic synchronization has a number of potential applications in laser physics, chemical reactor, secure communication, biological network, power systems, etc. It is well-known that the key idea of synchronization becomes the use of the master system output to control the slave system so that the output of the response system is capable of asymptotically tracking the master system output. In the past two decades, a number of control design techniques have been developed for the chaotic control and synchronization. Of particular interest is the use of nonlinear control technique to achieve synchronization of chaotic systems. To the best of our knowledge, the synchronization problem with the help of nonlinear control strategies has attracted a great deal of attention such as variable structure control (Wang & Su, 2004), OGY method (Ott, Grebogi, & Yorke, 1990), adaptive control (Chen & Lu, 2002; Yu & Zhang, 2004), backstepping control (Park, 2006; Yassen, 2007; Krstic, Kanellakopoulos, & Kokotovic, 1995), observer-based control

(Mahboobi, Shahrokhi, & Pishkenari, 2006), dynamic surface control (Li, 2012), immersion and invariance control (Xie, Han, & Zhang, 2012; Xie, Han, & Chen, 2013), sliding mode control (Zhang, Ma, & Liu, 2004), and so on.

Although considerable research has addressed the chaotic synchronization via the above-mentioned nonlinear control approaches, less attention has been devoted to synchronization of chaotic systems via the interconnection and damping assignment passivity-based control (IDA-PBC) scheme. There have not recently been any research works addressing the synchronization problem based on the IDA-PBC strategy.

In the past decade, the IDA-PBC methodology has been one of the most popular design methods for nonlinear control (Ortega, Castanos, & Astolfi, 2008; Ortega, Van der Schaft, Maschke, & Escoba, 2002) and synchronization (Zhu, Zhou, Zhou, & Teo, 2012). This method has numerous advantages: it is a systematic methodology that ensures closed-loop stability, improves transient performance for underactuated mechanical systems (Zhu et al., 2012) and multi-machine power systems (Kanchanaharuthai, Chankong, & Loparo, 2015), and facilitates the determination of controller parameters as compared with the conventional control design strategies that include iterative tuning approaches. For example, the IDA-PBC design strategy facilitates any additional damping to the system via the selection of appropriate matrices, where each matrix includes the coupling between the electrical damping and the mechanical damping, thereby mitigating and suppressing the heavy oscillations in the multi-machine power systems (Kanchanaharuthai et al., 2015).

The paper continues this line of investigation and uses a technique based on the IDA-PBC scheme for the design of a nonlinear control law to accomplish chaotic synchronization. The proposed control law obtained is simple but efficient and easy to implement in practical applications. Besides, one can use only a single control to achieve chaotic synchronization. To evaluate the effectiveness of the proposed approach for chaotic synchronization, simulation studies are carried out on the Genesio system (Park, 2004) and Chua's circuit system (Zhou & Er, 2007), respectively.

The rest of this paper is organized as follows. The IDA-PBC method applied to synchronize the chaotic systems is provided in Section 2. In Section 3, two chaotic systems of particular interest are mentioned and the controller design for each chaotic system is given. In Section 4, simulation results are given. Finally, we conclude in Section 5.

2. IDA-PBC method

Interconnection and damping assignment - a formulation of Passivity-Based Control (PBC) - introduced by Ortega (Ortega & Garcia-Canseco, 2004) is a general method for the design of a high performance nonlinear controller for systems which can be described by a port-Hamiltonian model. This method not only assigns suitable dynamics to the closed-loop system, but being a Hamitonian formulation. It is also capable of providing a control design which achieves stabilization by rending the system passive with respect to a desired storage function and the injection of a suitable level of damping.

In this section, we present a brief recapitulation of the IDA-PBC method applied to the control of chaotic synchronization. The interested reader is referred to the survey and tutorial paper (Ortega & Garcia-Canseco, 2004) for more details, and in particular, (Kanchanaharuthai et al., 2015; Ortega, Galaz, Astolfi, Sun, & Shen, 2005; Galaz, Oreta, & Bazanella, 2003) for applications to transient stability of power systems, (Zeng, Zhang, Qiao, 2013) for power electronics applications, and so on.

Consider a nonlinear system that is affine in the control input u and whose dynamics is given by the following set of differential equations:

$$\dot{x}(t) = f(x) + g(x)u(x), \qquad (1)$$

with the state variables $x \in R^n$ and the control input $u \in R^m$. $f(x)$ and $g(x)$ are the smooth vector functions in the appropriate dimensions. The idea of IDA-PBC is to make the closed-loop system with a stabilizing (static) feedback control $u = \beta(x)$ as an explicit port-Hamiltonian system in the form:

$$\dot{x}(t) = (J_d(x) - R_d(x))\nabla H_d(x), \qquad (2)$$

where the matrices $J_d(x) = -J_d(x)$ and $R_d(x) = R^T(x) \geq 0$ denote the desired closed-loop interconnection structure and dissipation, respectively, which are determined by the designer, and a Hamiltonian function $H_d(x): R^{\tilde{n}} \to R$ is the desired total storage function for the closed-loop system that satisfies an equilibrium point $x_e = \arg\min H_d(x)$. $\nabla H_d(x) = \frac{\partial}{\partial x} H_d(x)$ is the gradient of $H_d(x)$.

In order to make (1) equal to (2), a solution of the so-called *matching equation* needs to be determined as shown below:

$$f(x) + g(x)\beta(x) = (J_d(x) - R_d(x))\nabla H_d(x) \tag{3}$$

with the matrices $J_d(x)$, $R_d(x)$, and $H_d(x)$ as design variables. The equilibrium point of the closed-loop system is stable at the origin if the desired Hamiltonian $H_d(x)$ is positive definite. Consequently, the time derivative of $H_d(x)$ along closed-loop trajectories becomes in the form

$$\frac{d}{dt} H_d(x) = -\nabla H_d^T(x) R_d(x) \nabla H_d(x) \le 0. \tag{4}$$

Therefore, $H_d(x)$ serves as a Lyapunov function for the closed-loop system and the origin is a stable equilibrium point. In (4), asymptotic stability can be guaranteed in a case that $R_d(x)$ is also strictly positive definite. In order to solve the matching equation in (3), we split (3) into two parts: a fully actuated part and an un-actuated part. Let $Q(x)$ be $\begin{bmatrix} g^{\perp}(x) & g^{\dagger}(x) \end{bmatrix}^T$ where $g^{\perp}(x)$ denotes a full-rank left annihilator of $g(x)$, i.e. $g^{\perp}(x)g(x) = 0, \operatorname{rank}(g^{\perp}(x)) = \tilde{n} - \tilde{m},$ while $g^{\dagger}(x)$ denotes a left inverse of $g(x)$, i.e. $g^{\dagger}(x)g(x) = I$. After multiplying (3) from the left by $Q(x)$, we obtain a partial differential equation (PDE) and an algebraic equation, respectively, as follows.

$$g^{\perp}(x) f(x) = g^{\perp}(J_d(x) - R_d(x))\nabla H_d(x) \tag{5}$$

$$\beta(x) = g^{\dagger}(x)[(J_d(x) - R_d(x))\nabla H_d(x) - f(x)] \tag{6}$$

From the PDE in (5), it is noted that $J_d(x)$ and $R_d(x)$ are free to be chosen by the designer with the constraint of skew-symmetry and positive semi-definiteness, respectively. $H_d(x)$ may be totally or partially fixed, if we can ensure that the Lyapunov stability conditions are satisfied: namely, (i) $\nabla H_d(x_e) = 0$, (ii) $\nabla^2 H_d(x_e) > 0$. Thus, the functions $J_d(x)$, $R_d(x)$, and $H_d(x)$ need to be determined such that (5) is satisfied with the Hamiltonian having an isolated minimum at the desired equilibrium point $x_e \in R$ so that the equilibrium is stable and the Lyapunov stability conditions are also satisfied. In addition, x_e is in the largest invariant set under the closed-loop dynamics (2) which is contained in $\{x \in R^{\tilde{n}} \mid \nabla H_d^T R_d \nabla H_d = 0\}$. An estimate of the attraction domain for this closed-loop system is then given by the largest bounded level set $\{x \in R^{\tilde{n}} \mid H_d \le c\}$.

As a result, the control law $u = \beta(x)$ can be directly computed from (6) as follows.

$$\begin{aligned} \beta(x) &= [g^T(x) g(x)]^{-1} g^T(x) \\ &\quad \times \{[J_d(x) - R_d(x)] H_d(x) - f(x)\} \end{aligned} \tag{7}$$

The key step in this design method is the solution of the PDE that guarantees stability of the closed-loop system. This technique relies on the concept of exact model matching of the closed-loop system with a certain desired behavior that is determined by the pre-specified interconnection structure and dissipation matrices. Roughly speaking, the control objective of this technique is to find a stabilizing control law which can ensure that this closed-loop system behaves exactly like the pre-specified target system achieving exact model matching. In order to solve the matching equation above, there are different approaches to solve the PDE (5) as follows.

- Algebraic IDA: when the desired energy function is assigned and selected a priori, then PDE (3) becomes an algebraic equation in $J_d(x)$ and $R_d(x)$. Eventually, the controller is designed using (7).
- Non-parameterized IDA: $J_d(x)$ and $R_d(x)$ are selected to accomplish the desired structure of the closed-loop system; subsequently, all assignable energy functions compatible with that structure are characterized. This characterization is

provided in terms of a solution of the PDE (3). In addition, among the family of solutions, we choose the one with an equilibrium point x_e.

The overview of these and other approaches (Ortega & Garcia-Canseco, 2004) along with the applications to different examples (Dorfler, Jonhsen, & Allgower, 2009) can be investigated further in details.

3. Synchronization of chaotic systems

Consider the drive chaotic system as follows:

$$\dot{x} = f(x), \tag{8}$$

where $x \in R^n$ denotes the state vector and $f(x)$ is an $n \times 1$ matrix. In contrast, the response system is in the form of

$$\dot{y} = g(y) + u, \tag{9}$$

where $y \in R^n$ denotes the state vector and $g(y)$ is an $n \times 1$ matrix. Let $e = y - x$ be the synchronization error vector.

The aim of this section is to show how to design a state-feedback controller via an interconnection and damping assignment passivity-based control (IDA-PBC) method (Ortega & Garcia-Canseco, 2004; Ortega et. al., 2005; Galaz et al., 2003) for chaotic synchronization such that the trajectory of the response system (9) with an initial condition y_0 can approach the drive system (8) with an initial condition x_0 and eventually achieve the following synchronization requirement:

$$\lim_{t \to +\infty} |e(t)| = \lim_{t \to +\infty} |y(t, y_0) - x(t, x_0)| = 0, \tag{10}$$

where $|\cdot|$ denotes the Euclidean norm.

In two chaotic system applications of interest, the non-parameterized IDA approach and the algebraic IDA approach are used to find the proposed controllers that are capable of achieving the desired chaotic synchronization requirement, namely the synchronization between the chaotic drive (master) system and the controlled response (slave) system as well as leading to the closed-loop error signals converging to zero. Throughout this work, our investigation is under the following assumption: all parameters of drive and controlled response systems are precisely known and the proposed controllers are designed based on those known ones.

To show the effectiveness of the IDA-PBC approach applied on a wide variety of chaotic systems, the Genesio system and the Chua's circuit system are used as case studies with the details as follows.

3.1 Genesio system

Consider the drive Genesio system as follows.

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -cx_1 - bx_2 - ax_3 + x_1^2 \end{cases} \tag{11}$$

where $x_i (i = 1, 2, 3)$ are the state variables. a, b, c denote the positive real constants. In addition, the aim of this section is to design a state feedback controller u such that the controlled response systems of the forms:

$$\begin{cases} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= y_3 \\ \dot{y}_3 &= -cy_1 - by_2 - ay_3 + y_1^2 + u \end{cases} \tag{12}$$

is asymptotically synchronized with the drive Genesio system (11). By subtracting (11) from (12), we have the error dynamics as

$$\begin{cases} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= e_3 \\ \dot{e}_3 &= -ce_1 - be_2 - ae_3 + y_1^2 - x_1^2 + u \end{cases} \tag{13}$$

where $e_i = y_i - x_i, i = 1, 2, 3$. Therefore, our objective is to design an IDA-PBC controller u for the closed-loop error dynamics (13) such that the error signals between drive system (11) and the controlled response system (12) converge to zero and thus both systems are asymptotically synchronized.

In order to apply the IDA-PBC method, we can write the system (13) in the general form (1), i.e., $\dot{e} = f(e) + g(e)u$ as follows.

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} e_2 \\ e_3 \\ -ce_1 - be_2 - ae_3 + y_1^2 - x_1^2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

Proposition 1: For any initial values, the drive chaotic system (11) can asymptotically synchronize with the controlled response system (12) and the equilibrium point x_e of the closed-loop error dynamics (13) is asymptotically stable with the static state feedback controller in a form:

$$\begin{aligned} u = J_{23}e_2 - r_3(-e_1 + \Phi'(e_1, e_3)) \\ + ce_1 + be_2 + ae_3 - y_1^2 + x_1^2, \end{aligned} \quad (14)$$

where $\Phi(e_1, e_3) = \frac{1}{2}(e_3 + e_1 J_{23})^2$. J_{23} and $r_3 > 0$ are an arbitrary constant and a free parameter, respectively. In addition, for the Hamiltonian form (2) the desired interconnection matrix $J_d(e)$ and the damping matrix $R_d(e)$ are selected as shown in (13) along with the following energy function in a form:

$$H_d(e) = -e_1 e_3 - \frac{1}{2}e_1^2 J_{23} + \Phi(e_1, e_3) + \frac{1}{2}e_2^2 \quad (15)$$

Proof: Based on non-parameterized IDA approach, we fix the interconnection matrix $J_d(e)$ and the damping matrix $R_d(e)$ as

$$J_d(e) = -J_d(e) = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & J_{23} \\ 0 & -J_{23} & 0 \end{bmatrix}, \quad R_d(e) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & r_3 \end{bmatrix} \geq 0, \quad (16)$$

where J_{23} with $r_3 > 0$ are free parameters.

Following an idea given in (Shen, Sun, Ortega, & Mei, 2005), we can show that the energy function can be determined using the matrix structures in (16).

From this choice of those matrices, it is obvious that the PDE (5) characterizing the admissible energy functions are in the form:

$$\begin{cases} \dfrac{\partial H_d}{\partial e_2} = e_2 \\ -\dfrac{\partial H_d}{\partial e_1} + J_{23}\dfrac{\partial H_d}{\partial e_3} = e_3 \end{cases} \quad (17)$$

By using a commercial symbolic language software, e.g. Maple, all admissible functions $H_d(x)$ are obtained as in (17) where $\Phi(e_1, e_3)$ is an arbitrary differentiable function and has to be selected such that each energy function $H_d(x)$ has an isolated minimum at the desired equilibrium point $x_e = (0,0,0)$. This implies from LaSalle's invariance principle that

$$e_1 = 0 \Rightarrow y_1 = x_1, e_2 = 0 \Rightarrow y_2 = x_2, \quad e_3 = 0 \Rightarrow y_3 = x_3.$$

Also, the controlled response system (12) synchronize the drive system (11) by the proposed controller, u. Thus, the resulting control law u can be straightforwardly obtained from Proposition 1. This completes the proof.

3.2 Chua's circuit system

Consider the Chua's circuit system as follows.

$$\begin{cases} \dot{z}_1 &= p_1(z_2 - z_1 - f(z_1)) \\ \dot{z}_2 &= z_1 - z_2 + z_3 \\ \dot{z}_3 &= -p_2 z_2 \end{cases} \quad (18)$$

where z_i $(i = 1,2,3)$ are the state variables. $f(z_1) = p_4 z_1 + \frac{1}{2}(p_3 - p_4)(|z_1 + 1| - |z_1 - 1|)$. It is obvious that the circuit considered is not the strict feedback form (Krstic et al., 1995; Zhou, & Er, 2007); thus, it can be transformed into the desired feedback form from selecting new variables (Zhou & Er, 2007) as follows.

$$x_1 = z_3,\ x_2 = z_2,\ \text{and}\ x_3 = z_1 \quad (19)$$

Subsequently, we obtain the drive Chua's circuit system in the strict feedback form, which allows us to directly employ the IDA-PBC technique as follows.

$$\begin{cases} \dot{x}_1 &= -p_2 x_2 \\ \dot{x}_2 &= x_3 + x_1 - x_2 \\ \dot{x}_3 &= f(x) \end{cases} \quad (20)$$

where

$$f(x) = p_1 x_2 - p_1(1 + p_4)x_3 - \frac{1}{2}p_1(p_3 - p_4)(|x_3 + 1| - |x_3 - 1|).$$

Similarly, the aim of this section is to design a state feedback controller u such that the controlled response system of Chua's circuit system of the form:

$$\begin{cases} \dot{y}_1 &= -p_2 y_2 \\ \dot{y}_2 &= y_3 + y_1 - y_2 \\ \dot{y}_3 &= f(y) + u \end{cases} \tag{21}$$

with

$$f(y) = p_1 y_2 - p_1(1+p_4) y_3 \\ -\frac{1}{2} p_1(p_3 - p_4)(|y_3+1| - |y_3-1|),$$

is asymptotically synchronized with the drive system of Chua's circuit (20). By subtracting (20) from (21), we obtain the error system as

$$\begin{cases} \dot{e}_1 &= -p_2 e_2 \\ \dot{e}_2 &= e_3 + e_1 - e_2 \\ \dot{e}_3 &= f(x,y,e) + u \end{cases} \tag{22}$$

where $e_i = y_i - x_i \ (i = 1,2,3)$ and

$$f(x,y,e) = p_1 e_2 - p_1(1+p_4)e_3 - \frac{1}{2} p_1(p_3 - p_4) \\ \times(|y_3+1| - |y_3-1|) - |x_3+1| + |x_3-1|)$$

.

Therefore, our objective is to design an IDA-PBC controller u for the system (22) such that the error signals between the drive system and the controlled response system of Chua's circuit eventually approaches zero, leading to the fact that both systems are asymptotically synchronized. For this system, the Algebraic IDA approach is utilized to show the effectiveness of IDA-PBC strategy as described below.

Proposition 2: With the aid of the Algebraic IDA approach, the chaotic drive system (20) can be asymptotically synchronized by the controlled response system (21) for any initial values and the equilibrium point x_e of the error system (22) is asymptotically stable with the static state feedback controller

$$u = -J_{23} p_2 e_2 - r_3 \gamma_3 e_3 - f(x,y,e). \tag{23}$$

Moreover, for the Hamiltonian form (2), the desired interconnection and damping matrices $(J_d(e)$ and $R_d(e))$ in (25) can be determined using the following energy function:

$$H_d(e) = \frac{1}{2}(\gamma_1 e_1^2 + \gamma_2 e_2^2 + \gamma_3 e_3^2), \\ \gamma_i > 0, \ i = 1,2,3. \tag{24}$$

Proof: Based on the Algebraic IDA approach, we firstly choose the desired energy function as $H_d(x)$ in (24); subsequently, the PDE in (5) becomes an algebraic equation in $J_d(e)$ and $R_d(e)$ with the following structure:

$$J_d(e) = -J_d(e) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & J_{23} \\ 0 & -J_{23} & 0 \end{bmatrix},$$

$$R_d(e) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{bmatrix} \geq 0 \tag{25}$$

where J_{23} are directly calculated from (5) and $r_i > 0, \ i = 2,3.$

After some simple calculation, we obtain a PDE that becomes the algebraic equation of the form:

$$\begin{cases} -\dfrac{\partial H_d}{\partial e_2} = -\gamma_2 e_2 = -p_2 e_2 \\ \dfrac{\partial H_d}{\partial e_1} - r_2 \dfrac{\partial H_d}{\partial e_2} + J_{23} \dfrac{\partial H_d}{\partial e_3} = \gamma_1 e_1 - r_2 \gamma_2 e_2 + J_{23} e_3 = e_3 + e_1 - e_2 \end{cases} \tag{26}$$

Therefore, it is easy to obtain the following results.

$$-\gamma_2 e_2 = -p_2 e_2 \Rightarrow \gamma_2 = p_2 \\ \gamma_1 e_1 = e_1 \Rightarrow \gamma_1 = 1 \\ -r_2 \gamma_2 e_2 = -e_2 \Rightarrow r_2 \gamma_2 = 1 \\ J_{23} \gamma_3 e_3 = e_3 \Rightarrow J_{23} = \frac{1}{\gamma_3} \tag{27}$$

From the analysis above, it follows that from selecting $J_d(e)$ and $R_d(e)$ in (25), the closed-loop error system consisting of (25) and (23) matches the model (2). Consequently, the time derivative of the energy function along the trajectories of (3) satisfies the following equality:

$$\dot{H}_d(e) = -\nabla^T H_d(e) R_d(x) H_d \\ = -\gamma_2 e_2^2 - \gamma_3 e_3^2 \leq 0.$$

It can be concluded from using LaSalle's invariance principle that the equilibrium point x_e of the error system (19) is asymptotically stable. Consequently, the controlled response and drive systems will approach synchronization for any initial values. This completes the proof.

4. Simulation results

In this section, the IDA-PBC controller methodology is applied on the Genesio system as well as Chua's circuit system to verify and demonstrate the effectiveness of the proposed method. To evaluate the effectiveness of the proposed controller, the simulation results using the proposed (IDA-PBC) controller are compared with an existing nonlinear (BSP-backstepping) control scheme, which is explained partially in the Appendix (Krstic et al., 1995).

Additionally, the closed-loop performances of the systems are evaluated by using computer simulations. The complete system dynamics are obtained by solving the differential equations (11)-(13) and (20)-(22) in the MATLAB environment. The time domain simulations are carried out to investigate the performances of the designed controllers (u) for such systems.

- **Genesio system:** for the simulation, we assume that the initial conditions, $(x_1(0), x_2(0), x_3(0)) = (2, -3, 1)$ and $(y_1(0), y_2(0), y_3(0)) = (-2, 3, -5)$ are used. Consequently, the initial value $(e_1(0), e_2(0), e_3(0))$ of the error dynamics is $(-4, 6, -6)$. In order to investigate a chaotic behavior, three parameters are selected as $a = 1.2,\ b = 2.92,\ c = 6$ and the parameters of the control law as $J_{23} = 1.5,\ r_3 = 1.8$. With the proposed IDA-PBC scheme the chaotic synchronization of the system (9) and (8) as well as the resulting error system (10) are shown in Figures 1 and 2 where both Figures illustrate the synchronization errors and the state trajectories of this system, respectively.
- **Chua's circuit system:** in the simulation, we assume that the initial conditions, $(x_1(0), x_2(0), x_3(0)) = (0.2, 0.5, 0.3)$ and $(y_1(0), y_2(0), y_3(0)) = (2, 0.3, 0.4)$, are employed. Thus, the initial value $(e_1(0), e_2(0), e_3(0))$ of the error dynamics is $(1.8, -0.2, 0.1)$. In order to show a chaotic behavior, four parameters of the circuit are $p_1 = 0.5, p_2 = p_4 = 1, p_3 = 5$ and the parameters of the control law as $J_{23} = 1/\gamma_3 = 1.5, r_3 = 1.8$. With the proposed IDA-PBC technique the chaotic synchronization of the system (21) and (20) as well as the resulting error system (22) are illustrated in Figures 3 and 4, showing the system errors and the system state variables, respectively.

From the simulation results above, it is obvious that the IDA-PBC design technique is capable of not only achieving the synchronizations of two chaotic systems (the synchronization errors converge to zeros), but also accomplishing better dynamic performances (the transient responses for the closed-loop error dynamics of two chaotic systems can be improved) as compared to backstepping control (BSP) methods.

5. Conclusions

In this paper, the synchronization problems for some chaotic systems, in particular Genesio and Chua's circuit systems, have been investigated. With the aid of the IDA-PBC scheme, a nonlinear control law for asymptotic chaotic synchronization has been proposed. Even if the proposed control law is easy to implement in practical applications, it is still effective. Finally, the simulations of two chaotic systems are presented to illustrate the effectiveness, feasibility, and validity of our proposed scheme. Besides, they provide better transient responses and synchronization errors than the backstepping control strategy.

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7. Appendix

A. Genesio system with a backstepping controller

For any initial values, the chaotic drive system (11) can be asymptotically synchronized by the controlled response system (12) and the equilibrium point x_e of the closed-loop error dynamics (13) is asymptotically stable with the backstepping (BSP) controller in the form:

$$u = -w_2 - c_3 w_3 + \frac{\partial \alpha_2}{\partial w_1}\dot{w}_1 + \frac{\partial \alpha_2}{\partial w_2}\dot{w}_2 + ce_1 + be_2 + ae_3 - y_1^2 + x_1^2 \qquad (A.1)$$

where

$$\begin{cases} w_1 = e_1, w_2 = e_2 - \alpha_1(w_1), \\ w_3 = e_3 - \alpha_2(w_1, w_2), \\ \alpha_1(w_1) = c_1 w_1, \\ \alpha_2(w_1, w_2) = -w_1 + \frac{\partial \alpha_1}{\partial w_1}\dot{w}_1 - c_2 w_2. \end{cases} \qquad (A.2)$$

In this system the tuning parameters are chosen as $c_1 = 0.1, c_2 = c_3 = 1,$ so that the closed-loop error system is asymptotically stable.

B. Chua's circuit system with a backstepping controller

For any initial values, the chaotic drive system (20) can be asymptotically synchronized by the controlled response system (21) for any initial values and the equilibrium point x_e of the error system (22) is asymptotically stable with the backstepping (BSP) controller in the form:

$$u = -w_2 - c_3 w_3 + \frac{\partial \alpha_2}{\partial w_1}\dot{w}_1 + \frac{\partial \alpha_2}{\partial w_2}\dot{w}_2, + f(x, y, e) \qquad (B.1)$$

where

$$\begin{cases} w_1 = e_1, w_2 = e_2 - \alpha_1(w_1), \\ w_3 = e_3 - \alpha_2(w_1, w_2), \\ \alpha_1(w_1) = c_1 w_1, \\ \alpha_2(w_1, w_2) = (p_2 - 1)w_1 + (1 - c_2)w_2 \\ \qquad + \alpha_1(w_1) + \frac{\partial \alpha_1}{\partial w_1}\dot{w}_1 \end{cases} \qquad (B.2)$$

In this system, the tuning parameters are chosen as $c_1 = 0.25,\ c_2 = 0.5, c_3 = 1,$ so that the closed-loop error system is asymptotically stable.

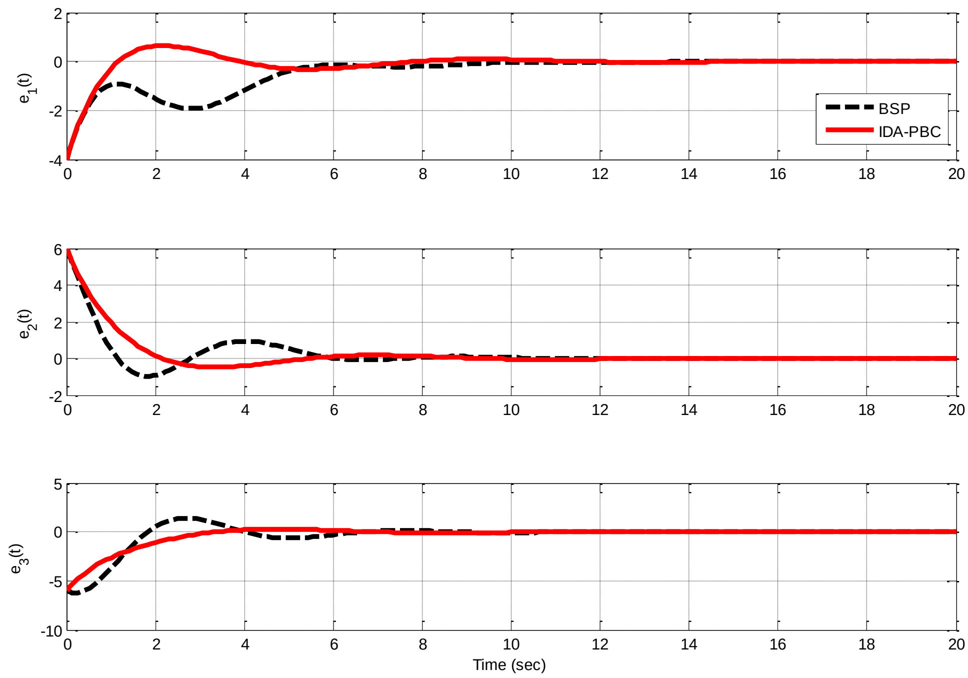

Figure 1 Synchronization errors $(e_1(t), e_2(t), e_3(t))$ of Genesio system

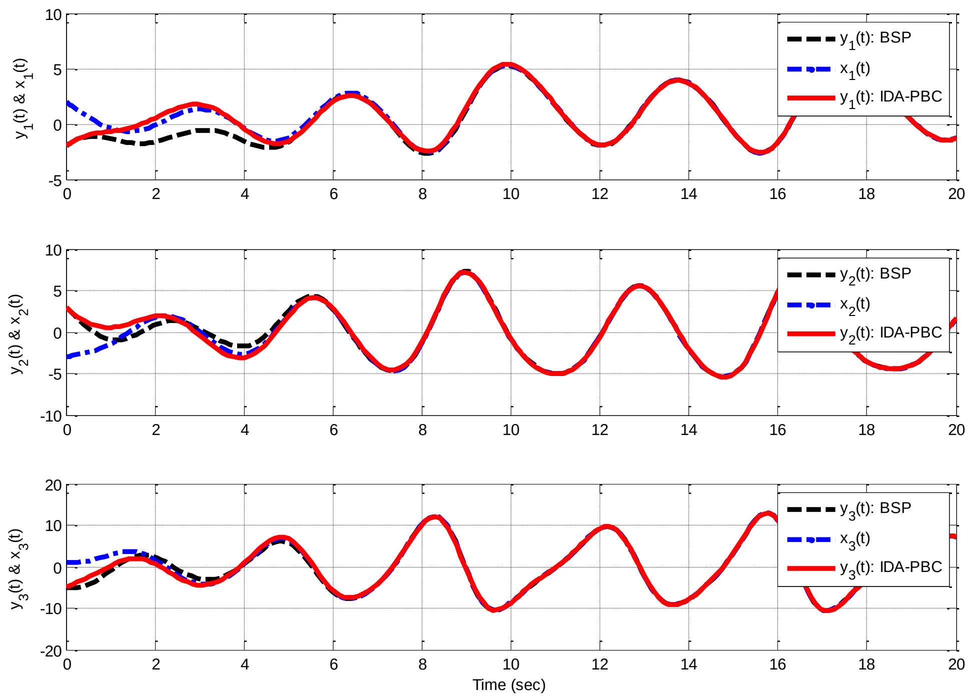

Figure 2 State trajectories of $y_i(t)$ and $x_i(t)$ $(i = 1,2,3)$ of Genesio system

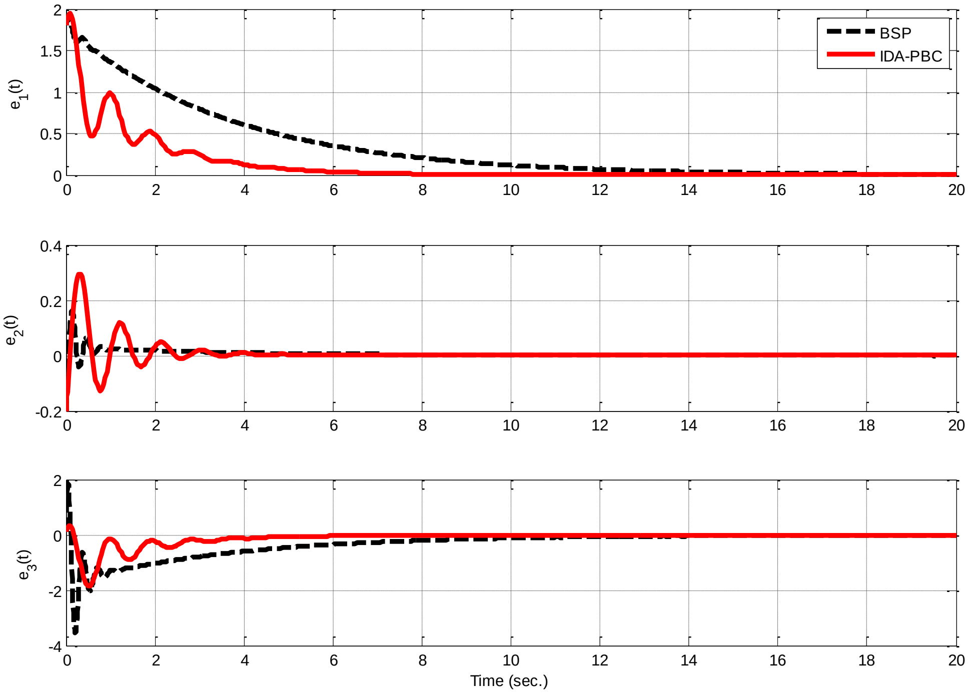

Figure 3 Synchronization errors $(e_1(t), e_2(t), e_3(t))$ of Chua's circuit system

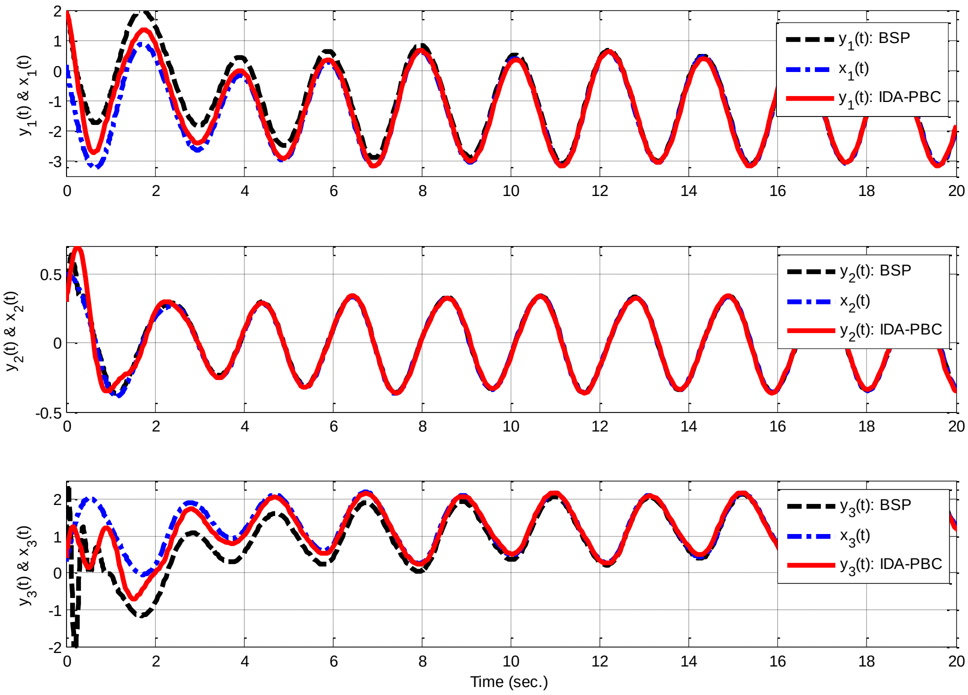

Figure 4 State trajectories of $y_i(t)$ and $x_i(t)$ $(i = 1, 2, 3)$ of Chua's circuit system