Influence of pair breaking effects on the long-range odd triplet superconductivity in a ferromagnet/superconductor bilayer

Thanasit Rachataruangsit[1] and Suthat Yoksan[2]*

[1]Physics Department, Faculty of Science, Burapha University, Chonburi 20131
E-mail: boonlit@buu.ac.th
[2]Physics Department, Faculty of Science, Srinakharinwirot University, Bangkok 10110
E-mail: suthat@swu.ac.th

*Corresponding Author

Submitted 30 May 2011; accepted in final form 27 November 2011

Abstract

The spin-dependent potential together with magnetic impurity and the spin-orbit scatterings were incorporated into the de Gennes-Takahashi-Tachiki theory of a diffusive superconductor-and-ferromagnetic metal to derive a formulation of the odd triplet superconductivity proximity effect. It is found that when the spin exchange interaction is inhomogeneous, i.e., the Neel spiral magnetic order, a new type of triplet condensate is generated, due to the broken time-reversal invariance. The triplet amplitude still contains the s-wave state, similar to the conventional singlet pairing, but the frequency symmetry must be odd to obey the Pauli's exclusion principle. As a result, the self-consistent order parameter contains only the singlet pair amplitude. The superconducting critical temperature of the bilayer is obtained in the single mode approximation and takes the form of the Abrikosov-Gorkov formula. The necessary condition for the occurrence of the induced long-range triplet component in the ferromagnet layer is characterized by the modulation of the pair amplitudes in the transverse direction. The possibility of the cryptoferromagnetic state, corresponding to the finite value of the spiral wave vector, is demonstrated in favor of the superconductivity. In addition, the influence of the magnetic impurity and the spin-orbit scattering is to decrease the decay length and to increase the oscillation period of the pair amplitudes which in turn enhances the critical temperature but in a less pronounced non-monotonicity manner.

Keywords: *odd triplet superconductivity, ferromagnetism, proximity effect, transition temperature, impurity scattering, spin-orbit scattering*

1. Introduction

The great progress in nano-fabricated technology in the past decade has been paid to the study of hybrid heterostructures of different materials. In the sandwiched structures composed of a superconductor (SC) and a normal metal (N), the Cooper pairs can physically penetrate the N side in a monotonic decaying manner to a certain length and so the induced superconductivity can therefore occur in N. This phenomenon is called the proximity effect. If the N metal is substituted by a ferromagnetic metal (FM), the Cooper pairs can be affected by the influence of the magnetic ordering. This leads to an unusual behavior of the superconducting condensate in the FM region.

The most striking phenomena of the FM/SC system manifests itself in the Buzdin-Tagirov spin switch effect (Buzdin, 1999; Tagirov, 1999) and the Radovic pi-phase state (Radovic,1991) where the first one exists in a FM/SC/FM trilayer in which theories predicted that an antiparallel magnetization in the FM layers could provide the higher critical temperature than a parallel configuration (Baladie, 2001; Baladie, 2003; You, 2004; Krunavakarn, 2004; Halterman, 2005; Faure, 2007). While the latter phenomenon, occurs in SC/FM multilayers, demonstates that the phase shift of the pair amplitudes between the adjacent SC layers can take either the zero-phase or the pi-phase (Kuboya, 1998; Krunavakarn, 2006; Proshin, 2006; Barsic, 2007; Jiang, 1995; Jiang, 1996; Obi, 2005; Shelukhin, 2006).

The model of a homogeneous exchange interaction in FM has been widely used and predicts the pair amplitude function in the SC to penetrate into the FM for a short distance. However, if the magnetization is inhomogeneous, the situation will change drastically. A new type of the triplet condensate arises which results in a long range triplet component of Cooper pairs.

The objective of this study is to investigate the influence of the pair breaking effects and the inhomogeneous magnetization on

DOI : 10.14456/rjas.2011.14

the long-range triplet superconductivity, which has never been clarified until now. The motivation is to investigate the physics of the FM/SC bilayer with emphasis on the possible formation of the long-range triplet pair. In Section 2, we present a formulation of the generalized triplet proximity effect by extending the Takahashi-Tachiki (1986) theory within the de Gennes's correlation function approach (de Gennes, 1966), so the spin-orbit scattering and the magnetic impurity scattering can be included in a phenomenological way (Auvil, 1989). The obtained Usadel transport-like equations are found to agree perfectly with those of Champel and Eschrig (2005) in the absence of the pair breaking scatterers. In Section 3, we study the model of the inhomogeneous Neel type spiral exchange field in the FM/SC bilayer and solve the critical temperature (T_C) equation as a function of a rotating spiral wave vector and an in-plane momentum which represents the modulation of the pair amplitude in the transverse direction. Finally, conclusions are presented in Section 4.

2. General formalism

We use the generalized de Gennes-Takahashi-Tachiki theory to derive the FM/SC odd triplet proximity effect. According to the de Gennes' correlation function formalism, the motion of the normal electrons at temperature T is controlled by the diffusion process. Suppose the system is characterized by the position-dependent material parameters such as the density of states $N(r)$, the diffusion coefficient $D(r)$, and the phonon mediated electron-electron pairing interaction $V(r)$ in the s-wave channel. Near the second order phase transition the superconducting order parameter $\Delta(r)$ in the s-wave state takes the linearized form

$$\Delta(\vec{r}) = \pi T N(\vec{r}) V(\vec{r}) \sum_{\omega} \int d^3 s Q_{\omega}(\vec{r},\vec{s}) \Delta(\vec{s}) \qquad (1)$$

where $\omega = (2n+1)\pi T$ with $n = 0, \pm 1, \pm 2, \ldots$, and T is the temperature.

The scalar kernel $Q_{\omega}(\vec{r},\vec{s})$ is related to the matrix kernel $\hat{Q}_{\omega}(\vec{r},\vec{s})$ by the relation

$$Q_{\omega}(\vec{r},\vec{s}) = \frac{1}{2} Tr \hat{Q}_{\omega}(\vec{r},\vec{s}) \qquad (2)$$

where Tr means the diagonal summation.

The equation of motion for the matrix kernel in the diffusive regime is governed by the equation

$$\left(|\omega| + \frac{1}{\tau_{\omega}} - \frac{1}{2} D \Pi^2\right) \hat{Q}_{\omega} + \frac{i}{2}(\vec{I} \cdot \hat{\sigma} \hat{Q}_{\omega} + \hat{Q}_{\omega} \vec{I} \cdot \hat{\sigma}) +$$

$$\frac{1}{\tau_{\omega}}\left(\hat{Q}_{\omega} - \sigma_2 \hat{Q}_{\omega}{}^{tr} \sigma_2\right) = \delta(\vec{r} - \vec{s}) \qquad (3)$$

In the above equation, the gauge invariant operator $\vec{\Pi} = \nabla - (2ie/c)\vec{A}$ describes the electromagnetic coupling through the vector potential $\vec{A}$, the spin exchange interaction $\vec{I} \cdot \hat{\sigma}$ represents the electron spins $\hat{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ coupling to the vector exchange field $\vec{I}(\vec{r})$. The scattering rates due to the magnetic impurity τ_m and the spin-orbit term τ_{so}, are calculated by averaging the Matsubara Green's functions over the impurity sites in the Born approximation, and δ is the delta function.

The Usadel transport-like equation can be obtained from (3) by defining the matrix pair amplitude

$$\hat{F}(\vec{r}, \omega) = \int d^3 s \hat{Q}_{\omega}(\vec{r},\vec{s}) i\sigma_2 \Delta(\vec{s}) \qquad (4)$$

However, it is useful to expand the matrix pair amplitude along the superfluid pairing states

$$\hat{F}(\vec{r}, \omega) = \left[F_s + \vec{F}_t \cdot \hat{\sigma}\right] i\sigma_2 \qquad (5)$$

where F_s and $\vec{F}_t = (F_{tx}, F_{ty}, F_{tz})$ are respectively the scalar spin singlet and the vector spin triplet. Inserting (4) into (3) and separating along the pairing basis, using (5), we then have the scalar and the vector equations

$$\left(|\omega| + \frac{1}{\tau_m} - \frac{D}{2}\Pi^2\right) F_s + i\vec{I} \cdot \vec{F}_t = \Delta(\vec{r}) \qquad (6)$$

$$\left(|\omega| + \frac{1}{\tau_m} - \frac{D}{2}\Pi^2 + \frac{2}{\tau_{so}}\right) \vec{F}_t + i\vec{I} F_s = 0 \qquad (7)$$

The obtained set of equations (6) and (7) are the Usadel transport-like equations (Usadel, 1970) which include the perturbation effects on the superconducting phase, namely, the orbital diamagnetism, the vector exchange field, the magnetic impurity and the spin-orbit scattering.

To discuss the pairing symmetry, we write down (1) in the form

$$\Delta(\vec{r}) i\sigma_2 = \pi T N(\vec{r}) V(\vec{r}) \sum \hat{F}(\vec{r}, \omega) \qquad (8)$$

Therefore $\hat{F}$ contains the antisymmetric $i\sigma_2$ and symmetric $\hat{\sigma} i\sigma_2$ matrices. According to the Pauli's exclusion principle, the spatial functions Δ and F_s will have an even symmetry whereas $\vec{F}_t$ possesses an odd symmetry. So $\hat{F}(\vec{r})$ does not satisfy the condition $\hat{F}(-\vec{r}) = \hat{F}(\vec{r})$. Nevertheless by imposing the even spatial parity of $\hat{F}(\vec{r}, \omega)$ and taking the even symmetry of $F_s(\vec{r}, \omega)$ and the odd frequency symmetry of $F_t(\vec{r}, \omega)$ into account, a new type of the spin triplet pair amplitude arises, and is the so called Berezinskii's odd frequency triplet condensate (Berezinskii, 1974). Thus (8) becomes

$$\Delta(\vec{r}) = \pi T N(\vec{r}) V(\vec{r}) \sum F_s(\vec{r}, \omega) \qquad (9)$$

which still possesses a conventional form and explain why there is only the spin singlet pairing responsible for the superconducting phase.

In applications to the layered structures, the materials parameters in each layer are treated separately which means that the pairing functions are connected by the boundary conditions at the interfaces

$$\xi_s \nabla \hat{F}^s = \gamma \xi_f \nabla \hat{F}^f, \hat{F}^s = \hat{F}^f - \gamma_b \xi_f \hat{n} \cdot \nabla \hat{F}^f \qquad (10)$$

and at the outer surfaces

$$\nabla \hat{F}^{s,f} = 0 \qquad (11)$$

The superscript s(f) refers to the SC(FM) layer,

the diffusion length $\xi_{s(f)} = \sqrt{D_{s(f)}/2\pi T_{c0}}$

where T_{c0} is the isolated superconducting critical temperature. The parameter γ is the resistivity mismatch between the two layers in contact. The boundary resistance γ_b characterizes the pair jumping from SC to FM in the direction outward normal to the interface.

3. Critical temperature of FM/SC bilayers

Assuming that the dirty-limit conditions are held and there is no external magnetic field applying to the system, so the FM/SC proximity problems are well described by the generalized Usadel equations (6) and (7) without the vector potential, and (9).

To calculate the critical temperature of a FM/SC bilayer we model two infinite slabs located in the y-z plane so the x-axis represents the direction of the pair amplitude propagation from SC $(0 < x < d_s)$ to FM $(-d_f < x < 0)$. The inhomogeneous exchange field inside FM is the spiral magnetic order that rotates in an in-plane with a spiral wave vector Q,

$$\vec{I}(y) = I(0, \sin Qy, \cos Qy) \qquad (12)$$

Within this configuration, the Q=0 limit corresponds to the homogeneous fixed exchange field which has the spin magnetization vector pointing constantly along the z-direction. Therefore the crypto-ferromagnetic model corresponds to a finite Q value.

Using the standard method it can be shown that an analytical expression of the reduced critical temperature $t_c = T_c/T_{co}$ takes the form like the Abrikosov-Gorkov (1961) formula which reads

$$\ln t = \Psi\left(\tfrac{1}{2}\right) - \Psi\left(\tfrac{1}{2} + \tfrac{1}{2t}\left[\left(p\xi_s\right)^2 + \tfrac{W_f(Q)}{d_s/\xi_s}\right]\right) \qquad (13)$$

Where $\Psi(x)$ is the digamma function, $t = T/T_c$
And $W_f(Q)$ is the boundary function that contains all information involved in the system,

$$W_f(Q) = \frac{\left(\eta_{p+} + \eta_{p-}\right) B_{fp+} B_{fp-}}{\eta_{p+} B_{fp-} + \eta_{p-} B_{fp+} + \zeta^2 N_{fp}} \qquad (14)$$

with

$$N_{fp} = \frac{\eta_{p+}B_{fp+} + \eta_{p-}B_{fp-}}{\eta_{p+}\eta_{p-}} - \frac{\left(\eta_{p+} + \eta_{p-}\right)B_{fp+}B_{fp-}}{\eta_{p+}\eta_{p-}B_{fp0}}$$

and

$$B_{fpj} = \frac{\gamma}{\gamma_b + \coth\left(k_{fpj}d_f\right)/k_{fpj}\,\xi_f}$$

The spectrum $k_{fp}^2 = \left(k_{fp0}^2, k_{fp\pm}^2\right)$ consists of the long-range mode k_{fp0}^2 and the short-range ones $k_{fp\pm}^2$, namely

$$k_{fp0}^2 = \frac{1}{\pi T_{c0}\xi_f^2}\left(\frac{1}{\tau_m} + \frac{2}{\tau_{so}}\right) + p^2 + Q^2$$

$$k_{fp\pm}^2 = \frac{1}{\pi T_{c0}\xi_f^2}\frac{1}{\tau_m} + p^2 \pm \frac{2i}{\xi_I^2}\eta_{p\mp}$$

For the long-range mode, the spectrum $k_{fp\pm}^2$ does not depends on the exchange length, $\xi_I = \sqrt{D_f/I}$ provides that the pair amplitudes having a long penetration length in a decay manner of the order of the characteristic length $1/k_{fp0}$. The pair breaking scattering rates τ_m and τ_{so} as well as the in-plane momentum p and the spiral wave vector Q give an identical feature, in reducing the decay length $1/k_{fp0}$. For the short-range mode, the spectrum $k_{fp\pm}$ is a complex quantity,

$$\eta_{p\pm} = \sqrt{1 - \zeta^2 - \eta_{so}^2}$$

with $\eta_{so} = \eta + 1/I\tau_{so}, \eta = \left(Q\,\xi_I\right)^2/4$,

therefore the corresponding characteristic length $1/k_{fp\pm}$ contains the penetration length and the oscillation length given by the real part and the imaginary part, respectively.

At this stage we summarize the obtained results as follows; (i) the inhomogeneous exchange field and the in-plane momentum are the important ingredients for the appearance of the induced long-range triplet, (ii) the magnetic impurity scattering flips the spins of paired electrons, in both the singlet and the triplet states, so the penetration lengths are reduced, and (iii) the spin-orbit scattering displays two distinct features simultaneously, it gives an extra decay of the long-range mode, while for the short-range modes the exchange field strength is dissipated by coupling with the spin-orbit interaction as a result the oscillation period of the pair amplitude is increased.

4. Conclusion

We have presented a formulation of the odd triplet superconductivity proximity effect within the de Gennes correlation function approach, and applied it to study the behavior of the diffusive FM/SC bilayered structure. The spin-orbit interaction and the magnetic impurity scattering are also incorporated into the Usadel transport-like equation. We found that the spin-orbit interaction tends to break the singlet pair indirectly through the exchange interaction by mixing it with the triplet pair, while the magnetic impurity flips all spin states. The long-range triplet pair correlation which is defined as the non-zero spin projection on the spin quantization axis is found to be generated in the presence of the inhomogeneous exchange field. We use the model of the Neel type spiral magnetic order that rotates in the plane and has no contribution in the perpendicular direction to the interface. The linearized self-consistent order parameter equation has been solved approximately in the single mode, and the critical temperature equation is derived to determine the dependence of the critical temperature T_c on the spiral wave vector Q and the ferromagnetic layer thickness d_f. The inhomogeneous superconducting state is characterized by the in-plane modulating wave vector p. We have shown that the induced long-range triplet arises when the pair amplitudes are modulated spatially in the transverse direction.

5. Acknowledgements

This work is supported by grants from the Thailand Research Fund (TRF) and the Commission on Higher Education (CHE).

6. References

Abrikosov, A.A., & Gorkov, L.P. (1961). Contribution to the theory of superconducting alloys with paramagnetic impurities. *Sov. Phys. JETP*, 12(6), 1243-1253.

Auvil, P.R., Ketterson, J.B., & Song, S.N. (1989). Generalized de Gennes-Takahashi-Tachiki proximity effect theory. *J. Low. Temp. Phys*, 74(1/2), 103-117.

Baladie, I., Buzdin, A.I., Ryzhanova, N., & Vedyayev, A.V. (2001). Interplay of

Buzdin, A.I., Vedyayev, A.V., & Ryzhanova, N.V. (1999). Spin orientation dependent superconductivity in S/F/S structures. *Europhys. Lett*., 48(6), 686-691.

Barsic, P.H., Valls, O.T., & Halterman, K. (2007). Thermodynamic and phase diagrams of layerd superconductor/ferromagnet nanostructures. *Phys. Rev. B*, 75, 104502.

Champel, T., & Eschrig, M. (2005). Effect of an inhomogeneous exchange field on the proximity effect in disordered superconductor-ferromagnet hybrid structures. *Phys. Rev. B*, 72, 054523.

de Gennes, P.G. (1966). *Superconductivity of Metals and Alloys*, New York, USA: Benjamin.

Faure, M., Buzdin, A.I., & Gusakova, D. (2007). On the theory of ferromagnet/ superconductor heterostructures. *Physica C*, 454, 61-69.

Halterman, K., & Valls, O.T. (2005). Nanoscale ferromagnet-superconductor-ferromagnet switches controlled by magnetization orientation. *Phys. Rev. B*, 72, 060514(R).

Jiang, J.S., Davidovic, D., Reich, D.H., & Chein, C.L. (1995). Oscillatory superconducting transition temperature in Nb/Gd multilayers. *Phys. Rev. Lett*., 74(2), 314-317.

Jiang, J.S., Davidovic, D., Reich, D.H., & Chein, C.L. (1996). Superconducting transition in Nb/Gd/Nb trilayers. *Phys. Rev. B*, 54(9), 6119-6121.

Krunavakarn, B., Sritrakool, W., & Yoksan, S. (2004). Nonmonotonic critical temperature in ferromagnet/ superconductor/ferromagnet trilayers. *Physica C*, 406, 46-52.

Krunavakarn, B., & Yoksan, S. (2006). Upper critical fields of ferromagnet/

superconductivity and magnetism in superconductor/ferromagnet structures. *Phys. Rev. B*, 63, 054518.

Baladie, I., & Buzdin, A.I. (2003). Thermodynamic properties of ferromagnet/superconductor/ferromagnet nanostructures. *Phys. Rev. B,* 67, 014523.

Berezinskii, V.L. (1974). New model of the anisotropic phase of superfluid He3. *JETP Lett*., 20(9), 287-289.

superconductor layered structures. *Physica C*, 440, 25-34.

Kuboya, K., & Takanaka, K. (1998). Upper critical field and transition temperature of superconducting/ferromagnetic superlattices. *Phys. Rev. B*, 57(10), 6022-6028.

Obi, Y., Ikebe, M., & Fujishiro, H. (2005). Evidence for zero-and pi-phase order parameters of superconducting Nb/Co tri- and pentalayers from the oscillatory behavior of the transition temperature. *Phys. Rev. Lett*., 94, 057008.

Proshin, Y.N., Zimin, A., Fazleev, N.G., & Khusainov, M.G. (2006). Hierarchy of critical temperatures in four-layered ferromagnet/superconductor nanostructures and control devices. *Phys. Rev. B*, 73, 184514.

Radovic, Z., Ledvij, M., Dobrosavljevic-Grujic, L., Shelukhin, V., Tsukernik, A., Karpovski, M., Blum, Y., Efetov, K.B., Volkov, A.F., Champel, T., Eschrig, M., Lofwander, T., Schon, G., & Palevski, A. (2006). Observation of periodic pi-phase shifts in ferromagnet-superconductor multilayers. *Phys. Rev. B*, 73, 174506.

Tagirov, L.R. (1999). Low-field superconducting spin switch based on a superconductor/ ferromagnet multilayers. *Phys. Rev. Lett*, 83(10), 2058-2061.

Takahashi, S., & Tachiki, M. (1986). Theory of the upper critical field of superconducting superlattices. *Phys. Rev. B*, 33(7), 4620-4631.

Usadel, K.D. (1970). Generalized diffusion equation for superconducting alloys. *Phys. Rev. Lett*., 25(8), 507-509.

You, C.-Y., Bazaliy, Ya.B., Gu, J.Y., Oh, S.-J., Litvak, L.M., & Bader, S.D. (2004).

Magnetization-orientation dependence of the superconducting transition temperature calculated for F/S/F trilayer structures. *Phys. Rev. B*, 70, 014505.